CONJUGATE HEAT EXCHANGE IN THE INITIAL STAGE OF REFLECTION OF A VISCOUS SHOCK WAVE FROM A WALL

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A numerical solution of the conjugate heat-exchange problem for reflection of a normally incident viscous shock wave from a rigid wall is used to find the non-steady-state temperature of the wall and the thermal flux into the wall.

The problem of nonsteady-state heat exchange between a gas and a solid thermally conductive wall upon reflection from the latter of a viscous shock wave arises in the study of physicochemical properties of gases in shock tubes [1], ignition of solid fuels, and a number of other applications. Two stages can be distinguished in such a heat-exchange process. In the first (initial) stage, simultaneous formation of the reflected wave "front" and a thermal boundary layer in the gas near the wall surface occur. This process is significantly dependent on the thermal regime of the wall, which affects the further evolution of the flow [2]. Description of the gas flow in the first stage requires use of the complete Navier–Stokes equations. The second stage is characterized by the fact that the viscous structure of the reflected wave "front" is formed and the region of the wave "front" and the thermal boundary layer are completely separated. In this case development of the thermal boundary layer is related to the dynamics of the reflected "front" only through the boundary conditions on the outer edge of the layer. Asymptotic theory can then be used to describe heat exchange in the boundary layer [2, 3].

The initial stage of reflection of a viscous shock wave with uniform accompanying flow has previously been studied numerically [4, 5] in two limiting cases where either the wall temperature is constant or the thermal flux into the wall is assumed equal to zero. Heat exchange within the wall itself was not considered. In the present study the problem of gas heat exchange with the wall will be solved in the conjugate formulation and simultaneous evolution of temperature fields in the gas and wall will be studied. Such an approach is obviously more correct and not only allows consideration of "intermediate" heat regimes, but also permits establishment of the limits of applicability of the limiting formulations referred to above.

We will direct a Cartesian coordinate x along the normal to the wall in the direction of the gas, taking the origin at the surface. To describe the one-dimensional nonsteady-state motion of the compressed viscous thermally conductive gas during shock wave reflection from the wall, we use the system of equations:

$$\begin{gathered}\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}+\rho\frac{\partial u}{\partial x}=0,\\ \rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)=-\frac{\partial p}{\partial x}+\frac{4}{3}\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right),\\ \rho c_V\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}\right)=\frac{4}{3}\mu\left(\frac{\partial u}{\partial x}\right)^2-p\frac{\partial u}{\partial x}+\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right),\\ p=\rho RT,\ \mu=\mu_0\left(\frac{T}{T_0}\right)^n,\ \lambda=\lambda_0\left(\frac{T}{T_0}\right)^n.\end{gathered}\tag{1}$$

In the calculations the value n = 0.76 was used [6]. The value c_V is assumed constant. Wall heating is described by the thermal conductivity equation

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$$\rho_T c_T \frac{\partial T_T}{\partial t} = \lambda_T \frac{\partial^2 T_T}{\partial x^2}. \qquad (2)$$

System (1)-(2) will be considered with the following boundary conditions (on the wall surface conjugate conditions are imposed: equality of temperatures and thermal fluxes):

$$x = 0:\ u = 0,\ T = T_T,\ \lambda \frac{\partial T}{\partial x} = \lambda_T \frac{\partial T_T}{\partial x};$$
$$x \to +\infty:\ \frac{\partial u}{\partial x} = \frac{\partial \rho}{\partial x} = \frac{\partial T}{\partial x} = 0; \qquad (3)$$
$$x \to -\infty:\ \frac{\partial T_T}{\partial x} = 0.$$

The initial conditions are formulated in the following manner. At time t = 0 in the gas region (at x ≧ 0) "viscous" ρ, u, and T profiles are specified in the incident shock wave. The calculation method will be described below. In the wall (x ≦ 0) the temperature at time t = 0 is assumed constant and equal to the unperturbed gas temperature ahead of the incident shock wave.

For a numerical solution of the problem, in place of the coordinate x we introduce a mass Lagrangian coordinate ξ in the gas region and an extended coordinate η in the wall region:

$$\xi = \int_0^x \rho(x,\ t)\, dx,\quad \eta = -(c_T\rho_T/\lambda_T)^{1/2} x. \qquad (4)$$

Use of the Lagrangian coordinate ξ in the problem of wall heating by the shock wave has an important advantage. In the thermal boundary layer in the gas, which develops upon reflection of the shock wave, an abrupt cooling of gas layers occurs adjacent to the wall, which leads to a significant increase in gas density in this region [4]. As a result large gradients in parameter values (density and temperature) develop, these gradients being more significant, the slower the wall heats at the interphase boundary. It is obvious that in this case the nodes of the uniform calculation grid over the coordinate ξ will automatically be bunched near the wall in physical space and thus, the grid to a certain degree adapts to the solution obtained in the region of primary interest.

We will introduce the dimensionless variables

$$\bar{\xi} = \frac{\xi}{\mu_0}\left(\frac{p_0}{\rho_0}\right)^{1/2},\ \bar{t} = \frac{t p_0}{\mu_0},\ \bar{\eta} = \eta\left(\frac{p_0}{\mu_0}\right)^{1/2},\ \bar{\rho} = \frac{\rho}{\rho_0},$$
$$\bar{u} = u\left(\frac{\rho_0}{p_0}\right)^{1/2},\ \bar{p} = \frac{p}{p_0},\ \bar{T} = \frac{T}{T_0},\ \bar{\mu} = \frac{\mu}{\mu_0},\ \bar{\lambda} = \frac{\lambda\rho_0 T_0}{\mu_0 p_0}, \qquad (5)$$
$$\bar{\rho}_T = \frac{\rho_T}{\rho_0},\ \bar{T}_T = \frac{T_T}{T_0},\ \bar{c}_T = \frac{c_T\rho_0 T_0}{p_0},\ \bar{\lambda}_T = \frac{\lambda_T\rho_0 T_0}{\mu_0 p_0}.$$

The system of equations (1), (2) with boundary conditions (3) in the coordinates of Eq. (4) and dimensionless variables of Eq. (5) takes on the form (bar omitted)

$$\frac{\partial \rho}{\partial t} = -\rho^2 \frac{\partial u}{\partial \xi},$$
$$\frac{\partial u}{\partial t} = -T\frac{\partial \rho}{\partial \xi} - \rho\frac{\partial T}{\partial \xi} + \frac{4}{3}\mu\frac{\partial \rho}{\partial \xi}\frac{\partial u}{\partial \xi} + \frac{4}{3}\rho\frac{d\mu}{dT}\frac{\partial T}{\partial \xi}\frac{\partial u}{\partial \xi} + \frac{4}{3}\rho\mu\frac{\partial^2 u}{\partial \xi^2},$$
$$\frac{\partial T}{\partial t} = -(\gamma - 1)\rho\frac{\partial u}{\partial \xi}\left(T - \frac{4}{3}\mu\frac{\partial u}{\partial \xi}\right) + \qquad (6)$$
$$\frac{\gamma}{\mathrm{Pr}}\left[\mu\frac{\partial \rho}{\partial \xi}\frac{\partial T}{\partial \xi} + \rho\frac{d\mu}{dT}\left(\frac{\partial T}{\partial \xi}\right)^2 + \rho\mu\frac{\partial^2 T}{\partial \xi^2}\right],$$
$$\frac{\partial T_T}{\partial t} = \frac{\partial^2 T_T}{\partial \eta^2},\ p = \rho T,\ \mu = T^n,\ \lambda = \frac{\gamma T^n}{(\gamma - 1)\mathrm{Pr}};$$

$$\xi = \eta = 0: u = 0, \ T = T_{\text{T}}, \ \rho\lambda \frac{\partial T}{\partial \xi} = -\varepsilon \frac{\partial T_{\text{T}}}{\partial \eta};$$

$$\xi \to +\infty: \ \frac{\partial \rho}{\partial \xi} = \frac{\partial u}{\partial \xi} = \frac{\partial T}{\partial \xi} = 0, \ \eta \to +\infty: \ \frac{\partial T_{\text{T}}}{\partial \eta} = 0.$$

Here $\varepsilon = \sqrt{c_T \rho_T \lambda_T}$ is the dimensionless thermal activity of the wall material [7] (c_T, ρ_T, λ_T are dedimensionalized with Eq. (5)).

The initial $\rho(\xi)$, $u(\xi)$, and $T(\xi)$ profiles are calculated in the following manner. For a given incident shock wave Mach number M, steady-state Navier–Stokes equations for the compressible gas, written in an Eulerian cordinate system which moves at the constant velocity of the incident front, are used to determine parameter profiles in a steady-state viscous shock wave in a manner similar to that of [8]. The profiles thus obtained were recalculated for an Eulerian coordinate system fixed to the wall. Then a recalculation to the mass Lagrangian coordinate ξ is accomplished with the aid of the first relationship of Eq. (4), and the results dedimensionalized with Eq. (5). In the dimensionless variables the initial conditions have the form $t = 0$, $0 \leqq \eta < \infty$; $T_T = 1$.

The conjugate problem thus formulated, Eq. (6), was solved numerically by a spline-finite difference method. Approximation of functions and their derivatives in the time layer was accomplished with the aid of cubic splines, while time derivatives were approximated by finite differences. Such an approach has been proposed previously and used for solution of the Burgers equation [9, 10], which in a certain sense is a model of the one-dimensional nonsteady-state Navier–Stokes equations. Use of cubic splines gives fourth- and second-order approximations on the uniform grid for the first and second derivatives. On the nonuniform grid (used in the wall region) the order of approximation of the second derivative is maintained. It was shown in [9] using examples of the Burgers equation and a number of model hydrodynamics problems that the spline finite-difference method gives more accurate results than conventional three-point finite-difference methods. In the conjugate heat-exchange problem under consideration here the significant point is the formulation of the boundary condition on the wall surface for the first derivative of temperature, which in the final reckoning is used to calculate the thermal flux into the wall. The high order of approximation of the spatial derivatives is then an important advantage of the method. A special study of the accuracy of the spline finite-difference method using the problem of reflection of a viscous shock wave from a thermally insulated wall showed that not only the unknown functions, but also their first derivatives agreed well with an exact solution for the profiles in the reflected shock wave.

The calculation region in the gas was limited to the value ξ_∞, at which at the initial moment the dimensionless "viscous" parameters on the wall ($\xi = 0$) and at $\xi = \xi_\infty$ differ from their asymptotic values by less than 10^{-6} (for example, at $M = 2$, $\xi_\infty = 68.5$). The grid step $\Delta\xi$ was taken equal to ξ_∞/N. As a rule, in the calculations the value $N = 50$ was used. To monitor accuracy, individual realizations were recalculated with a step twice as small. The calculation region within the wall was limited to the coordinate $\eta_\infty = 25$-50. The first three $\Delta\eta$ steps from the interphase boundary were set equal to $\Delta\xi/4$, after which the step size was increased geometrically by a factor of 1.064. Determination of all derivatives in the time layer was accomplished by the drive method. Formulation of the boundary conditions for the corresponding difference equations on the external boundaries of the computation region presented no difficulty. On the phase boundary the temperature derivatives under conjugate conditions were approximated by unilateral expressions of fourth-order accuracy. Implicit and explicit methods were used for time. Calculations in the explicit method were performed in the following sequence: first the continuity equation was considered, then the energy equation, and finally the momentum equation, with the density and temperature values obtained in the subsequent time layer being used immediately in the remaining equations. The process of iteration over nonlinearity was organized similarly in the implicit methods. It developed that for the given problem the explicit method proved more economical, so it was used for all calculations.

Numerical results were obtained for $Pr = 0.75$, $\gamma = 1.4$, $M = 2$ and 5 with ε varied from 0 to $\sim 10^5$. Some principles of the heat-exchange process will be presented below for $M = 2$.

Figure 1 shows temperature profiles in the gas at various times for reflection of a shock wave from a wall with different values of the parameter ε. For comparison, reflected wave

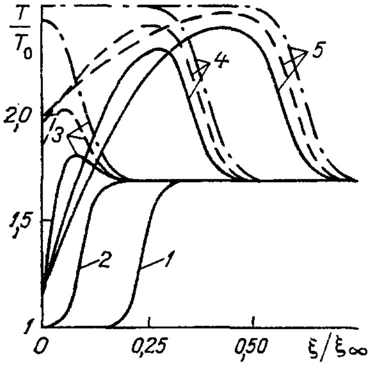

Fig. 1

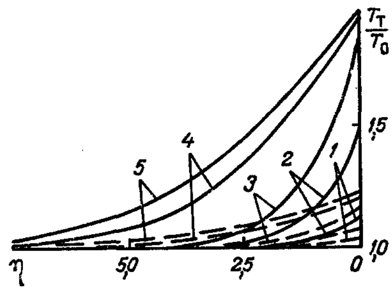

Fig. 2

Fig. 1. Temperature profiles in incident (1, t = 0; 2, 4.38) and reflected (3, t = 8.03; 4, 10.95; 5, 13.14) shock waves for various values of parameter ε: solid lines, ε = 75.8; dashes, 6.82; dash–dot, thermally insulated wall. t, ε, dimensionless.

Fig. 2. Temperature distribution in wall during shock wave reflection (1, t = 6.31; 2, 7.80; 3, 8.03; 4, 10.95; 5, 13.14) for various values of parameter ε: solid lines, ε = 6.82; dashes, 75.8; η, t, ε, dimensionless.

parameters are shown for an adiabatic wall. In the incident wave the temperature profiles are independent of ε and all lines (solid, dashed, and dash–dot) coincide at t = 0 and t = 4.38.

It is evident from Figs. 1 and 2 that in the process of shock-wave reflection the quantity ε has a strong effect on the temperature field evolution in both the gas and the wall. With increase in ε there is a more intense delay in formation of the reflected shock-wave viscous structure and cooling in the departure of the reflected front as compared to the case of the adiabatic wall. A similar difference in the behavior of the gas near the wall for the limiting models of an ideally thermally conductive wall and a thermally insulated wall was established in [4]. As is evident from Fig. 2, the thickness of the heated layer in the wall increases rapidly with time, although in the variables used at a given t it is only weakly dependent on ε.

For the ε values shown in Figs. 1 and 2, the process of thermal interaction of a shock wave with a wall differs markedly from the limiting cases, where instead of conjugate conditions on the phase boundary an adiabatic state or constant temperature is assumed. In both limiting cases the process of wall heating was not considered in studying the evolution of the flow in the gas. It was assumed that heat transfer in the wall could be described by the thermal conductivity equation with boundary conditions on the surface for temperature or thermal flux found by solution of the gas dynamic problem. Choice of a wall thermal regime obviously depends on the values of the parameters defining the problem. In the given problem one of the most important defining parameters is the dimensionless thermal activity of the wall material ε. In connection with this, it is of interest to determine the ranges of ε over which the simpler limiting formulations can be employed instead of the conjugate problem. To study this question we turn to an examination of the time dependence of wall surface temperature and thermal flux into the wall for various values of ε.

As is evident from Fig. 3a, the temperature on the phase boundary increases abruptly to some value and then remains practically constant. This result is in qualitative agreement with the experimental data of [11], in which it was found that after a discontinuous constant over a long period of time, including the second stage of the reflection process in which the viscous shock "front" and the thermal boundary layer in the gas are completely separated. It follows from Fig. 3 that at fixed t the quantity T_w decreases monotonically, while q_w increases monotonically with increase in ε. At $\varepsilon \gtrsim 10^3$ the functions $T_w(t)$ and $q_w(t)$ are practically independent of ε. Calculations performed also show that at $\varepsilon \gtrsim 10^3$ the wall temperature is practically constant and equal to its initial value (over the time interval considered it increases no more than 5%) so that from the viewpoint of gas dynamics, instead of conjugate conditions one can consider the problem with constant wall temperature (boundary conditions of the first sort) while in calculating heat transfer one can specify

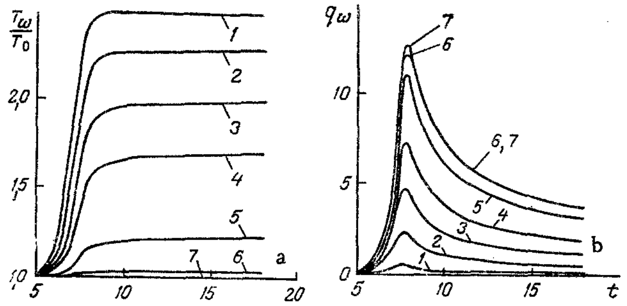

Fig. 3. Wall temperature (a) and thermal flux into wall (b) vs time for various values of parameter ε: 1) ε = 0.341; 2) 2.41; 3) 6.82; 4) 15.15; 5) 75.8; 6) 758; 7) 7580; q_w referred to $p_0\sqrt{p_0/\rho_0}$.

the thermal flux on its surface (boundary condition of the second sort), calculated from the gas temperature gradient on the wall. On the other hand, at $\varepsilon \lesssim 0.4$ evolution of the flow in the gas practically coincides with the evolution of a flow upon reflection of a shock wave from an adiabatic wall, while the wall temperature behind the reflected front differs by less than 5% from the value corresponding to the ideal theory of shock wave reflection [12]. In this case the thermal flux into the wall is close to zero over the entire time interval (see, for example, curve 1 of Fig. 3b) and to study the gas flow during shock wave reflection the condition $\partial T/\partial \xi = 0$ (boundary condition of the second sort) can be used on the phase boundary in place of the conjugate condition. In this case heat transfer in the wall can be calculated with only a temperature boundary condition (boundary condition of the first sort), which is obtained on the phase boundary for reflection of a shock wave from an adiabatic wall.

Calculations performed for M = 5 show that the character of all the dependences considered above is maintained unchanged. The quantitative estimates for the ranges of the dimensionless parameter ε in which the limiting formulations with type I and II boundary conditions may be used also remain in force. This result permits a more solidly grounded specification of the wall thermal regime in calculating nonsteady state heat exchange during reflection of a normally incident shock wave.

NOTATION

x, Cartesian coordinate; t, time; u, ρ, p, T, μ, λ, velocity, density, pressure, temperature, viscosity, and thermal conductivity of gas; c_p, c_V, specific heat of gas at constant pressure and temperature; R, ideal gas constant; M, incident shock wave Mach number; ρ_T, c_T, λ_T, density, specific heat, and thermal conductivity of wall material; T_t, wall temperature; T_w, temperature on phase boundary; q_w, thermal flux from gas to wall. Subscripts: 0, unperturbed value ahead of incident shock wave; ∞, limit of calculation region; $Pr = c_p\mu/\lambda$; $\gamma = c_p/c_V$.

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EFFECT OF VISCOSITY ON THE VORTEX STRUCTURE OF A FLOW AROUND A CYLINDER AND THE DRAG OF THE CYLINDER WITH AND WITHOUT A DISK IN FRONT OF IT

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Large-scale vortex structures appearing in a flow around a cylinder with and without a disk for Reynolds numbers from 40 to 2500 are studied numerically and experimentally.

The use of detached flows in different practical applications, in particular, to form a forward detached zone by placing in front of a blunt body a diskotic attachment in order to reduce the head drag of the body [1], has stimulated the study of large-scale vortex structures, arising near poorly streamlined bodies, based on physical experiments in a hydrodynamic tube and computer calculations. The combination of the methods of physical and numerical modeling enabled, on the one hand, obtaining more detailed information about the characteristic features of detached flow and the effect of geometric and flow parameters on the vortex structure, and on the other evaluating the reliability of the computational method used and its ability to describe correctly the basic features of the flow pattern as well as to predict with an accuracy adequate for practical applications its integral and local characteristics. The evolution of vortex structures for Reynolds number from 40 to 2500 is studied for the example of a uniform flow of an incompressible liquid around a cylinder of elongation λ (λ = 14) with and without a thin circular disk of radius r, placed coaxially in front of the flat face of the cylinder at a distance l. Here and below all geometric dimensions are scaled to the radius of the cylinder R. In this paper attention is devoted primarily to the study of the effect of convection and diffusion on vortex formation and the determination of the relationship between the structure of detached flow around bodies and their integral and local characteristics, in particular, the head drag of blunted bodies. The velocity U and the density ρ of the incident flow are used as scaling parameters. The radius of the connecting rod is constant and equal to 0.07.

In the experimental study of an axisymmetric, low-velocity, uniform flow of air around a disk—semiinfinite-cylinder arrangement [2], performed at high Reynolds number Re of the order of 10^5, it was found that there exist optimal geometrical dimensions r_{opt} and l_{opt}, corresponding to minimum profile drag of the body. Thus, for example, for r_{opt} = 0.75 and l_{opt} = 0.75 the profile drag coefficient of a body with this shape C_{xp} = 0.02, i.e., it is close to the drag coefficient of a body with a conveniently streamlined shape. In this paper, bodies of revolution of the same configuration with optimal or close to optimal dimensions at high Reynolds numbers in a laminar flow are studied. It is especially interesting to compare the role of vortex formation in the reduction of drag of systems of poorly streamlined bodies with the case of flow around individual bodies.

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